A numerical algorithm for decision-making in sustainable transport projects investment

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Abstract

In this paper, we develop a numerical algorithm that sets rules for investment in hydrogen infrastructure under both demand and costs uncertainties. In fact, the growth of the number of oil-fuled cars (which we call the demand) is a major source of uncertainty, as a high number of cars increases pollution and, on the other hand, makes a migration for a hydrogen transport necessary. In the same time, the investment cost is decreasing stochastically with time due to the worldwide R&D effort. We first develop a model, based on the IPAT method, to calculate the real cost of transport externalities. Using the real options method, we then show how to maximize inter-generational utility by choosing the optimal time to invest. We make use of a dynamic programming approach to develop an algorithm that gives, at each future moment, the thresholds for demand and cost for which it is better to invest. We calculate the expected waiting time until investing and show that we must wait a longer time before investing when the uncertainty is high.

1 Introduction

Transport is today fueled to a very large extent by oil. This is of great relevance from an environmental perspective, notably in view of climate change. Hydrogen has then been proposed as a long-term alternative for oil, as it is the "forever fuel", producing no harmful carbon dioxide emissions when burned and giving off as byproducts only heat and pure water. All that needs to be done is to extract hydrogen from various elements so that it is usable in fuel cells. Furthermore, the transition from a petroleum-based energy system to a hydrogen economy requires the construction of many new hydrogen plants and fueling stations, which involves large costs. Large R&D effort is then done to make the transition possible at a lower cost and to improve the lifetime of products, reduce costs and increase technical maturity.

In this paper, we consider a project consisting of building new hydrogen infrastructures under two kinds of uncertainties whose impact should to be correctly evaluated before any investment could be undertaken. In such circumstances, the decision makers have a tendency to delay such important investment which are highly uncertain. But delaying the investment is not always a good solution and we must, at a certain time, undertake the investment. In this work, we develop a method to determine whether it is optimal to invest at a given instant in a project whose cost is subject to uncertainty. Furthermore, we consider an additional uncertainty related to the demand of the project: The increase of the transport demand increases the number of cars and generates more externalities. We use the decision tree analysis to solve this problem and decide whether to invest or not. We also obtain, if the decision is to delay the project construction, the expected waiting time before investment. Note that the problem of investing in hydrogen motors was studied in [1], but only the cost uncertainty was considered, without taking into account the uncertainty related to the market demand. In our work in [4], we developed a closed-form solution for the population level for which it is optimal to invest in a project whose cost is constant and where the demand is growing following a geometric brownian motion. In this paper, we develop a general algorithm that helps making decisions in a sustainable transport project where both cost and demand grow stochastically. We also considered a more realistic model where the demand does not grow geometrically but remain limited.

2 Costs of transport environmental impacts

There is widespread scientific agreement that the increased concentrations of greenhouse gases (GHGs) are the consequence of human activities around the globe. Among these anthropogenic factors, the principal ones (often called "driving forces"') are (i) population, (ii) economic activity, (iii) technology, (iv) political and economic institutions, and (v) attitudes and beliefs [2].

Although there is a growing recognition of the important linkages between population and the environment, our understanding of exactly how these linkages operate is still rather limited. We may intuitively understand that human populations and their activities cause environmental change and that environmental change in turn affects the quality and condition of human lives, but the specific details of these interactions are still largely speculative. Population growth is widely regarded as an important cause of air pollution, and air pollution has shown to have serious adverse impacts on human health. Population growth causes air pollution, and air pollution reduce the rate of population growth. However the feedback is rather weak, since several studies suggest that urban air pollution may account for about %2 of all local death [3]. On the other hand, urban air pollution may have a larger impact on net migration than morality [3]. However, most of researchers have considered only the impact of population on pollution. In the next paragraphs we will present certain models that consider population as an actor of environmental treatement.

2.1 IPAT model description

The (IPAT) framework adopts the formulation: Impact= Population·Affluence·Technology. This framework, first proposed in the early 1970s by Ehrlich & Holdren as a part of a debate on the driving forces of environmental change, incorporates key features of human dimensions of environmental change into a model as follows:

$$I = P.A.T \tag{1}$$

where I is environmental impact, P is population, and A is affluence or economic activity per person. T is the environmental impact per unit of economic activity, which is determined by the technology used for the production of goods and services and by the social organization and culture that determine how the technology is mobilized.

The IPAT identity summaries a general relation between human behavior and its environmental impact. However, it can also be applied to a specific environmental issue. For example, I may represent an aggregate environmental indicator such as the amount of CO_2 emission. In this particular case, A corresponds to a specific behavior or consumption per capita which is a cause for the environmental impact represented by I. T stands for environmental impact per unit of the consumption or behavior represented by A.

Although this model is simple and integrates myriad effects into a single multiplier, it assumes *a priori* that the effects of P, A and T on I are strictly proportional and independent, which is not necessarily true: Empirical data are then used to find the exact formulation for each case. The following subsections present some applications of the IPAT model on specific environmental problems.

2.1.1 General ASIF equation for transport impacts

The IEA developed the ASIF equation based on the IPAT model to cover transport impacts [5]:

$$G = \sum_i A \cdot S_i \cdot I_i \cdot \sum_j F_{i,j}$$

where G is the emission of any pollutant summed over sources (modes) i; A is total travel activity, in passenger kilometers (or ton-km for freight), across all modes. S converts from total passenger (or freight) travel to vehicle travel by mode. I is the energy intensity of each mode (in fuel/passenger or tonne-km), and is related to the inverse of the actual efficiency of the vehicle, but it also depends on vehicle weight, power, and of course driver behavior and traffic. F is the fuel type j in mode i. In this model, the size of the population is present into the total travel activity A.

2.1.2 IPAT model for CO_2 emissions

In the special case of CO_2 emissions, the authors in [2] reformulated the model (1) slightly by the following:

$$I = aP^bA^ce \tag{2}$$

Here, I, P, and A remain, respectively, environmental impact, population, and affluence. The quantities a, b, c, and e can be estimated by applying standard statistical techniques. The coefficients b and c determine the net effect of population and affluence on impact, and a is a constant that scales the model. Technology is modeled as a residual term e. Their results show that the impact of population is roughly proportional to its size: This confirms the IPAT model and contradicts the views of those who are complacent about population growth. Furthermore, the coefficient b was estimated by 1,15, leading to an almost linear behavior of I regarding the population size P.

The author in [6] used this linear behavior to study the amount of carbon dioxide emission from car travel by an $I = PAT$ type decomposition equation, where the amount of the CO_2 emission is determined by population, car trip distance per person, occupancy rate, fuel efficiency, fuel structure and CO_2 intensity of fuel.

2.2 Pollution cost per person

The above analysis shows that the cost per polluter has almost a linear behavior. If D is the number of oil car owners, the total cost of emissions is calculated by the IPAT model by:

$$\text{Total Cost} \approx \text{Emission level} \cdot \text{Cost per tonne} = (D.A.T) \cdot (k) = k.(A.T).D$$

The pollution cost per polluter is then calculated by:

$$\text{Cost per polluter} = k.(A.T) \tag{3}$$

which is independent from the number of car owners D.

2.3 How to finance the projects using IPAT model?

Using the above-defined IPAT model, we are able to determine the real cost of the pollution. Taxation policies are then necessary in order to apply the principle called "polluter pays". The value calculated in Eqn. (3) is then transformed to taxes that polluters pay. This may include residential taxes (applied to the whole of the population) in the case of public transport, or taxes on oil or cars in the case of hydrogen infrastructure investment.

3 Sources of uncertainties

3.1 Uncertainty related to cost

In this work, we consider that the investment cost will decrease because of external, worldwide research and development (R&D) efforts in universities, government laboratories and companies. This corresponds for example to the case where the project consists of using hydrogen fuel [1]. This technological change cost is an exogenous process which can be modeled by the following jump process:

$$dC_t = -\Phi C_t dq$$

where $dq = 1$ with probability $1 - \lambda$ and $dq = 0$ with probability λ. For all $t > 0$, we note C_t the investment cost at time t, and $C_t \geq C_{min} = (1 - \Phi)^M C_0$ and Φ is a positive constant. M is the maximum number of downward states (the maximum times that the cost could go downward).

3.2 Uncertainty related to demand

Let we consider that D_t is growing stochastically according to :

$$D_t = D_0(K + (1 - K)e^{-\mu t})e^{-\frac{\sigma^2}{2}t + \sigma B_t} \quad X_0 = x \tag{4}$$

where μ, K and σ are strictly positive constants. We will call this kind of stochastic processes concave brownian motion (CBM).

The choice of this expression for the demand is not arbitrary, as our aim was to have a model where demand does not grows exponentially, but has a concave behavior. If D_0 is the population at $t = 0$, K can interpreted as a fraction of how much the population will be at long term. σ is the volatility parameter, and B_t is a standard brownian motion with zero mean and $\sqrt{t}$ standard deviation. The term $e^{-\frac{\sigma^2}{2}t}$ was added in order to have

$$E_{D_0}[D_t] = D_0(K + (1 - K)e^{-\mu t})$$

When $t \to \infty$, $E_{D_0}[D_t] \to D_0 K$. This is illustrated in Figure 1.

Using Itô calculations, we obtain:

$$dD_t = \mu(t)D_t + \sigma D_t dB_t \tag{5}$$

where

$$\mu(t) = \frac{-\mu(1 - K)e^{-\mu t}}{K + (1 - K)e^{-\mu t}} \tag{6}$$

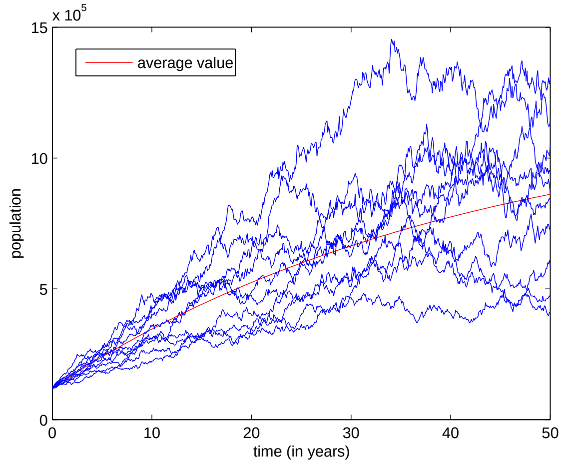

Figure 1: Examples of the demand growth ($\sigma = 0.24$,$\mu = 0.024$ and $K = 10$)

4 Decision tree analysis

4.1 Why using real options

We suppose that the agency disposes of an opportunity to invest in such a sustainable project during a certain time T. Hence the agency has the opportunity to invest at any time or to delay the investment until T. The decision to build such a project is irreversible and its infrastructure can not be used for any other purpose [7].

The objective of the public agency being to decrease the air pollution by constructing a sustainable transportation system, the classical decision method based on cost-benefit (Discount Cash Flow DCF) analysis is not suitable for the following reasons:

1. There are two sources of uncertainty: The cost of the project per city demander decreases stochastically over time, and the demand grows stochastically. DCF ignores the possibility of waiting for a better situation, and leads to values that do not exploit the option "invest now or later", whereas the real options method deals with the uncertainty about the future returns in a flexible way.

2. As detailed in [7], the uncertainty increases the firm's opportunity costs of investment and raises the threshold rate of return required to induce the firm to forgo its option to defer investment.

3. In contrast with the real options method, the DCF one ignores that the investment can be completely or partially irreversible and, in our case, the investment can be considered as completely irreversible. This assumption

of complete irreversibility is not very realistic, but the reversible part of such investment is negligible compared to the initial cost.

4. The analogy between the agency's opportunity to invest in a sustainable project and the holding of a financial call option argues for a real options approach. In fact, the firm has the right but not the obligation to buy an asset (the project) at a future, pre-determined time, at an exercise price (the total cost C, uncertain in our case).

We will apply then a real options approach to answer the following important question: until when is it preferable to delay the investment and how much is the value of this opportunity (option to defer)?

4.2 Dynamic programming approach

To evaluate this option we will use the dynamic programming technique (decision tree analysis). We recall that it consists in dividing the problem into two binary decisions: The immediate one and its generated value and the delaying one and its continuation value, at the end of the period T. Then by moving backward, and by repeating the same binary decision, we obtain the expected optimal time which lies in an expected interval in which the investment should be undertaken.

4.3 Utility of a polluter

Note that we are interested by the utility of a demander of the unsustainable transport, because the policy of internalization consists of billing polluters for the pollution they generate.

We consider the utility of a representative consumer (supposed neutral to risk):

$$U(c) = c \tag{7}$$

Let us now calculate the budget constraint of this representative polluter. The total cost caused by the unsustainable transport activities is given by IPAT's formula as described above. The budget constraint, consumption per head is then equal to:

$$c = m - \alpha_0 \tag{8}$$

So, we have the following indirect utility function

$$V_0 = U(c) = m - \alpha_0 \tag{9}$$

where m is the income per person per unit of time and $\alpha_1 = k.(A.T)$.

We suppose that the new project will result in lowering the pollution cost coefficient α from α_0 to α_1 where $\alpha_0 > \alpha_1$. If the total project cost is C is annualized over an infinite horizon, and shared equally among transport users.

The cost of the project per user per unit of time is then equal to $\frac{\rho C}{D}$, ρ being the discount rate, and D the number of transport users.

Once the project is under way, the budget constraint of a demander becomes

$$c + \alpha_1 = m - \frac{\rho C}{D_t}$$

From this new budget constraint, we can derive a new indirect utility function:

$$V_1(D_t) = c = m - \alpha_1 - \frac{\rho C}{D_t} \tag{10}$$

4.4 Net utility gain

If the public agency invests at time t, the net utility gain evaluated at t with an investment cost of C_m ($1 < m \leq M$) and a demand of d_t which is one of the realizations at time t of the process D_t is:

$$NU(d_t, C_m) = E_{d_t}(\int_t^{\infty} e^{-\rho(s-t)} D_s[V_1(D_s, C_m) - V_0(D_s)].ds) \tag{11}$$

where $V_0 = U(c) = m - \alpha_0$ and $V_1(D_s, C_m) = m - \alpha_1 - \frac{\rho C_m}{D_s}$. This gives

$$NU(d_t, C_m) = E_{d_t}(\int_t^{\infty} e^{-\rho(s-t)}[(\alpha_0 - \alpha_1)D_s - \rho C_m].ds) \tag{12}$$

$$NU(t, d_t, C_m) = (\frac{d_t(\alpha_0 - \alpha_1)}{K + (1-K)e^{-\mu t}})(\frac{K}{\rho} + \frac{(1-K)e^{-\mu t}}{\mu + \rho}) - C_m \tag{13}$$

Using the fact that:

$$E_{d_t}(D_s)) = d_t(\frac{(K + (1-K)e^{-\mu s})}{(K + (1-K)e^{-\mu t})}$$

The problem we face when calculating the net utility gain at time t is that we only know the value of the demand at time $t = 0$ ($X_0 = x$), its value at t being a random value. To calculate the net utility gain at time t, we must then explore all the possible values of the population at time t.

We begin by performing Monte Carlo simulations to explore the space of possible values at each time t. We then define the interval $[d_{min}, d_{max}]$ that includes most of the possible values (say 99%) of the population. We divide this interval into N subintervals, with subinterval $i \in [1, N]$ defined by:

$$[S_1(i) = d_{min} + (i-1)\frac{d_{max} - d_{min}}{N}, S_2(i) = d_{min} + i\frac{d_{max} - d_{min}}{N}]$$

Note that we choose a large N so that subintervals are small, each subinterval represents a possible state and we can represent subinterval i by the value $\bar{S}(i) = \frac{S_1(i)+S_2(i)}{2}$. We then define N values of the net utility gain at time t by:

$$NU(t, i, m) = (\frac{\bar{S}(i)(\alpha_0 - \alpha_1)}{K + (1-K)e^{-\mu t}})(\frac{K}{\rho} + \frac{(1-K)e^{-\mu t}}{\mu + \rho}) - C_m \tag{14}$$

4.5 Transition probabilities for the demand

The probability that, knowing that the demand is in interval i at time t, it will move to interval j at time $t+1$, is calculated by:

$$p_{i,j}(t) \approx Pr[D_{t+1} \in [S_1(j), S_2(j)] | D_t = \bar{S}(i)]$$

Using the formula:

$$dD_t = \mu(t)D_t + \sigma D_t dB_t$$

with $dB_t = \sqrt{dt}N(0,1)$, $dt = 1$. This gives

$$p_{i,j}(t) = Pr[N(0,1) \in [\frac{1}{\sigma \bar{S}_i \sqrt{dt}}(S_1(j) - \bar{S}_i - \mu(t)\bar{S}_i), \frac{1}{\sigma \bar{S}_i \sqrt{dt}}(S_2(j) - \bar{S}_i - \mu(t)\bar{S}_i)]]$$

Note that this can be expressed by the function $erfc(a) = \frac{2}{\sqrt{\pi}} \int_a^{\infty} e^{-x^2} dx$:

$$\begin{aligned} p_{i,j}^d(t) = \frac{1}{2}[&erfc(\frac{1}{\sigma \bar{S}_i \sqrt{2dt}}(S_1(j) - \bar{S}_i - \mu(t)\bar{S}_i)) \\ &-erfc(\frac{1}{\sigma \bar{S}_i \sqrt{2dt}}(S_2(j) - \bar{S}_i - \mu(t)\bar{S}_i))] \end{aligned}$$

Furthermore, the probability that the corresponding state of the demand at time t is i, is calculated by the following algorithm:

1. at $t = 0$:

$$P^d(0,i) = \begin{cases} 1 & \text{if } i = \left\lfloor \frac{N(d_0 - d_{min})}{d_{max} - d_{min}} \right\rfloor + 1 \\ 0 & \text{otherwise} \end{cases}$$

   $\lfloor y \rfloor$ being the largest integer less than y.

2. at $t > 0$ and for all state number j,

$$P^d(t,j) = \sum_{i=1}^{N} P^d(t-1,i) p_{i,j}^d(t-1)$$

4.6 Decision algorithm

Let $O(T,m,i) = \max[NU(t,i,m), 0]$ be the option value at time T if the investment cost is C_m and if the demand in the ith interval. At time T, the agency has two alternatives choices: Invest and get $NU(T,i,m)$ or never invest and get 0. By moving back one period, it gets:

$O(T-1,i,m) =$

$$\max[NU(T-1,i,m), \frac{\sum_{j=1}^{N} p_{i,j}^d(T-1) \times [\lambda O(T,j,m) + (1-\lambda)O(T,j,m+1)]}{1+\rho}]$$

At a time $t < T$, the agency has also two alternatives choices: Invest now and get $NU(t, i, m)$ or wait one period and then decide. At the next period, the investment cost will decrease with probability $1 - \lambda$ or remain the same with probability λ. We should then take the expected value of the option to wait actualized at time t. Let we denote that:

$$W(t, i, m) = \frac{\sum_{j=1}^{N} p_{i,j}^{d}(t) \times [\lambda O(t + 1, j, m) + (1 - \lambda) O(t + 1, j, m + 1)]}{1 + \rho}$$

More precisely, we can write the following algorithm:

- Start at the maturity date T at which a now or never decision should be undertaken.
- at $t = T$ the option is calculated as $O(T, i, m) = \max[NU(T, i, m), 0]$ for all $0 < m \leq M$ and $i \in [1, N]$.
- move back one period to $t = T - 1$ and calculate

$$O(T - 1, i, m) = \max[NU(T - 1, i, m), W(T, i, m)]$$

- move back one period and compute $O(T - 2, i, m)$, and so on until calculating $O(0, i, m)$.
- The first time t such that $NU(t-1, i, m) > W(t-1, i, m)$ and corresponding to a non-zero probability is then the optimal time to invest with C_m and with level of demand falls in state i. This happens when the value of the immediate investment is higher than the expected value of option to wait for this value of the investment cost and this level of demand.

4.7 Expected time to invest

We begin by introducing some definitions. A each time t, let $d(t)$ be the state of the demand and $c(t)$ be the state of the cost. Let us also define the following function:

$$G(t, i, m) = NU(t, i, m) - O(t, i, m) \tag{15}$$

The event A_τ="Investment occurred exactly at time τ" is equivalent to: B_τ ="Investment did not happened before τ" and H_τ="Demand at τ is high enough to invest and the cost of the project low enough". Note that we mean by the sentence: "Demand is high enough and the cost of the project low enough at time τ" that D_τ and C_τ authorize the investment, i.e. $NU(\tau, i, C_m) > W(\tau, i, m)$ or $G(\tau, d(\tau), c(\tau)) > 0$.

$$Pr[A_\tau] = Pr[B_\tau \cap H_\tau] = Pr[H_\tau | B_\tau] \times Pr[B_\tau]$$

The event B_τ is equivalent to: "Demand was not sufficient and cost was high for all $t < \tau$":

$$Pr[H_\tau | B_\tau] = Pr[G(\tau, d(\tau), c(\tau)) \geq 0 | G(t, d(t), c(t)) < 0 \forall t < \tau]$$

As the state of the system follows a Markov chain, this is equivalent to:

$$Pr[H_\tau|B_\tau] = Pr[G(\tau, d(\tau), c(\tau)) \geq 0 | G(\tau-1, d(\tau-1), c(\tau-1)) < 0]$$

$$= \frac{\sum_{(i,m):G(\tau-1,i,m)<0} P(\tau-1,i,m) \sum_{(j,l):G(\tau,j,l)\geq 0} p_{i,j,m,l}(\tau-1)}{\sum_{(i,m):G(\tau-1,i,m)<0} P(\tau-1,i,m)} \tag{16}$$

where $p_{i,j,m,l}(t)$ is the probability of changing the state from (i,m) to (j,l) at time t, given by:

$$p_{i,j,m,l}(t) = p^d_{i,j}(t) \times \begin{cases} \lambda & \text{if } l = m \\ 1-\lambda & l = m+1 \\ 0 & \text{otherwise} \end{cases}$$

and $P(t,i,m)$ is the probability that, at time t, the state is equal to (i,m):

$$P(t,i,m) = P^d(t,i) \times P^c(t,m)$$

(for $P^c(t,m)$ see section **??**).

Having obtained $Pr(H_\tau|B_\tau)$, we now return to the probability of event B_τ. It is calculated as follows:

$$Pr[B_\tau] = Pr[G(t, d(t), c(t)) < 0, \forall t < \tau]$$

$$= Pr[G(\tau-1), d(\tau-1), c(\tau-1)) < 0 | G(t, d(t), c(t)) < 0, \forall t < \tau - 1]$$

$$\times Pr[G(t, d(t), c(t)) < 0, \forall t < \tau - 1]$$

This is equivalent to:

$$Pr[B_\tau] = Pr[\bar{H}_{\tau-1}|B_{\tau-1}] \times Pr[B_{\tau-1}] = (1 - Pr[H_{\tau-1}|B_{\tau-1}]) \times Pr[B_{\tau-1}]$$

where $\bar{H}$ ="Population and cost are not sufficient at $\tau - 1$ "

This gives:

$$Pr[B_\tau] = Pr[B_{t_{min}}] \times \prod_{t=t_{min}}^{\tau-1} (1 - Pr[H_t|B_t])$$

where t_{min} is the first time at which investment is possible. As, by definition, $Pr[B_{t_{min}}] = 1$, then:

$$Pr[B_\tau] = \prod_{t=t_{min}}^{\tau-1} (1 - Pr[H_t|B_t]) \tag{17}$$

We propose then to use the following iterative algorithm in order to obtain the expected waiting time:

1. Begin at time t_{min}:

$$Pr[A_{t_{min}}] = Pr[H_{t_{min}}|B_{t_{min}}] = \sum_{(i,m):G(t_{min},i,m)\geq 0}^{N} P(t_{min}, i, m)$$

2. For all $t > t_{min}$, calculate $Pr[H_t|B_t]$ using Eqn.(16), and $Pr[B_t]$.

3. Beginning from $t_{min} + 1$ and incrementing the time until T, calculate:

$$Pr[A_t] = Pr[H_t|B_t]Pr[B_t]$$

4. The expected waiting time is calculated using:

$$\bar{w} = \sum_{t \geq t_{min}} Pr[A_t] \times t$$

4.8 Numerical applications

In this section, we examine the characteristics of the expected waiting time developed in the previous section and show how it depends on the values of the various parameters. Some numerical solutions will help to illustrate the results.

Figure 2 plots the expected waiting time as a function of the volatility. We can see that as the volatility increases we should wait more before investing. The volatility increases thus the waiting option value so that investing decision becomes more valuable under higher uncertainty.

In Figure 3, we can see that, as before, when the probability that the cost decreases each month increases, investment occurs sooner, as the cost will become rapidly low.

Finally, Figure 4 shows the expected waiting time as a function of interest rate ρ: The higher the interest rate the higher the annuity per person for the same level of population and total cost project which is shared equally between the people.

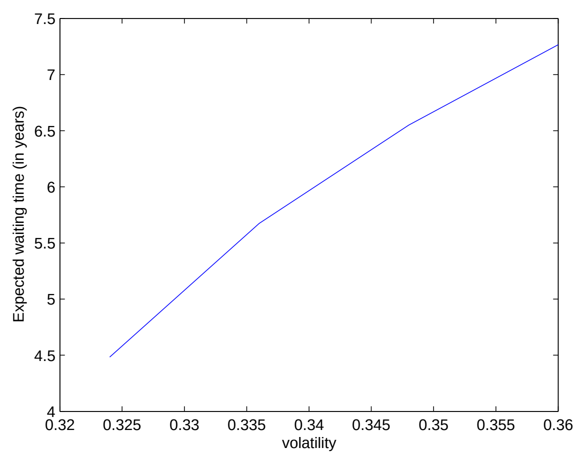

Figure 2: Expected waiting time function of the volatility ($\lambda = 0.99\%$,$\gamma(1) = 3 * 10^9$, $x_0 = 120000$, $K = 2$, $\rho = 7\%$, $\Phi = 0.05$)

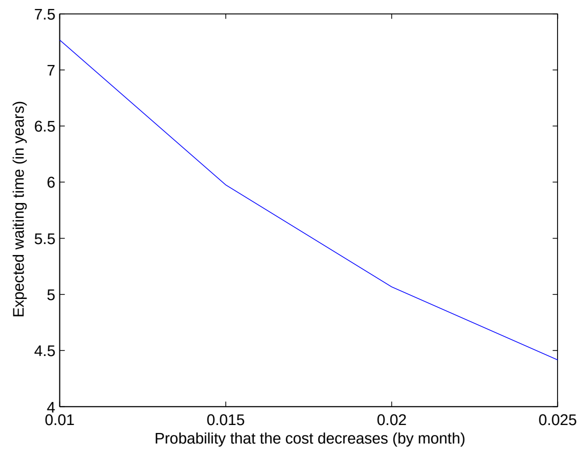

Figure 3: Expected waiting time function of the probability that the cost decreases ($\sigma = 0.36\%$,$\gamma(1) = 3 \times 10^9$, $x_0 = 120000$, $K = 2$, $\rho = 7\%$, $\Phi = 0.05$).

5 Remark about the upgrading cost

In Section 4.3, we considered a utility function, after the investment, of the form:

$$V_1(D_t) = c = m - \alpha_1 - \frac{\rho C}{D_t},$$

supposing that the cost of the project is divided over all demanders $\frac{\rho C}{D_t}$. This corresponds to a hydrogen infrastructure construction, where everybody must contribute to the project through taxes. However, a transition to a hydrogen transport involves also substituting the oil motor by a hydrogen motor, with a certain cost M_t that is paid entirely by the polluter. The utility function becomes then:

$$V_1(D_t) = c = m - \alpha_1 - \rho M - \frac{\rho C}{D_t} \tag{18}$$

M may also decrease stochastically with time ($M = M_t$), adding another source of uncertainty for the decision. This can be described, as for the infrastructure cost, by the following:

$$dM_t = -\Xi M_t dq$$

where $dq = 1$ with probability $1 - \delta$ and $dq = 0$ with probability δ.

The decision algorithm depends then on three random variables: The demand D_t, the motor cost M_t, and the infrastructure cost C_t. The net utility gain is then:

$$NU(t, S_i, C_m, M_k) = (\frac{\bar{S}(i)(\alpha_0 - \alpha_1 - \rho M_k)}{K + (1 - K)e^{-\mu t}})(\frac{K}{\rho} + \frac{(1 - K)e^{-\mu t}}{\mu + \rho}) - C_m \tag{19}$$

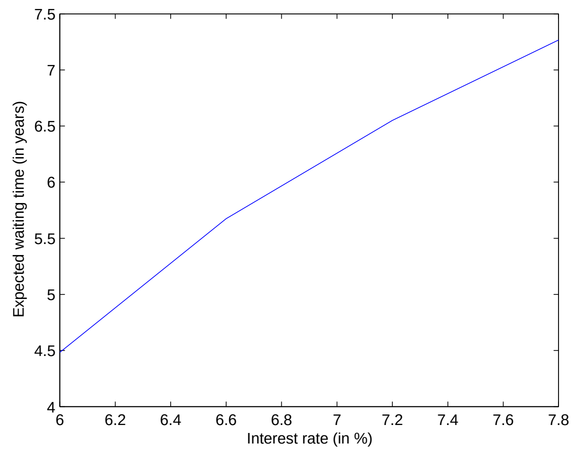

Figure 4: Expected waiting time function of the interest rate ($\sigma = 0.36\%$,$\gamma(1) = 3 \times 10^9$, $x_0 = 120000$, $K = 2$, $\lambda = 0.99$, $\Phi = 0.05$)

An backward algorithm, similar to that developed in Section 4.6, can be introduced with the additional random variable, and the expected waiting time can be obtained.

6 Conclusion

In the problem of investing in hydrogen fuel infrastructure, the effect of uncertainty is generally ambiguous because it affects both the benefits and the costs: Investing decreases pollution while waiting decreases project costs. We then used in this paper the real options method that is the most adapted decision-taking tool when uncertainty is high. We utilized of a dynamic programming approach when the investment is subject to two sources of uncertainty: Demand-growth and project-cost uncertainties. We determined, for each configuration corresponding to a demand level and a project cost, whether it is better to invest immediately or to delay the investment. We also calculated, for the case where delaying the investment is better, the expected waiting time before we can undertake the project construction.

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