

Access pricing under uncertainty: Margin squeezes, real options, and the cost of capital^{*}

P. N. Baecker^{*}, G. Grass, U. Hommel

*Department of Finance, Accounting, and Real Estate, European Business School,
International University Schloß Reichartshausen, D-65375 Oestrich-Winkel,
Germany*

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Abstract

Recent studies have emphasized the role of irreversibility, flexibility, and uncertainty in assessing the financial implications of regulated access pricing in the telecommunications market (Pindyck 2004, 2005). In particular it has been argued that mandatory unbundling of the local loop creates optionalities (real options), leading to a transfer of wealth from incumbents to entrants unaccounted for by a static investment perspective. We extend this line of research by explicitly considering the significance of margin squeezes, predatory pricing, and related pricing rules in a duopoly model tailored to the German context. Analytical and numerical methods are employed to characterize pricing behavior and market dynamics under demand uncertainty. We draw conclusions with respect to the incumbent's cost of capital under regulatory intervention.

Keywords: G31, G38, L96

JEL classification: telecommunications, real options, access pricing, margin squeezes, predatory pricing, cost of capital, option games

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^{*} Corresponding author.

E-mail addresses: philipp.baecker@ebs.edu (P. N. Baecker),
gunnar.grass@ebs.edu (G. Grass), ulrich.hommel@ebs.edu (U. Hommel).

1 Introduction

Existing approaches aimed at assessing the financial implications of regulatory intervention, by and large, take a static view on demand and thus fail to capture the impact of uncertainty and flexibility on value and risk. From the investor's perspective, they are unsuitable for making optimal decisions on substantial irreversible investments typically found in network industries such as telecommunications. From the regulator's perspective, they might bring about a misalignment of incentives and result in significant welfare losses.

The recent push for liberalization, most notably in the European telecommunications sector, together with technological advances, has provided new motivation for investigating the applicability of real option theory in the context of mandatory unbundling and other measures intended to promote competition and foster innovation in an industry that, by most accounts, can be described as a natural monopoly.

Although potential entrants, regulators, and researchers rightfully call for price controls in certain areas, neglecting the role of uncertainty in capital budgeting brings the risk of incorrectly predicting the incentives and economic impact of such instruments.

In this paper, we develop and apply a simple real option framework for assessing the impact of regulatory intervention on firm value and the cost of capital. In addition, a novel technique is presented, which facilitates the numerical derivation of myopically optimal policies in option games.

2 Previous Work

Among the first to address real options in a telecommunications context were Alleman and Noam (1999) and, more recently, Alleman (2002) as well as Alleman and Rappoport (2002, 2006). Closely related to this paper is the seminal work by Dixit (1991). In addition, Pindyck (2004, 2005a) provides a thorough analysis of mandatory unbundling from a real options perspective. For a discussion of real options and antitrust in general, consult Pindyck (2005b).

While debt financing and capital structure decisions are not the focus of this paper, it would be possible to extend the analysis to a richer setting incorporating such elements. Relevant contributions include, for example, Goldstein et al. (2001) and Miao (2005).

3 The Models

We describe four different models of a stylized telecommunications market, in which demand fluctuates randomly and an incumbent local exchange carrier faces competition by a new entrant. Due to the complexity of the problem at hand, we rely heavily on numerical methods to derive optimal policies and firm values.

3.1 Demand

Assume a finite time horizon $[0, T]$. Further assume an inverse demand function

$$P(t) \equiv Y(t) \cdot D(Q(t)) \equiv \left(\frac{1}{Q(t)} \right)^\eta, \quad (1)$$

where $\varepsilon \equiv 1/\eta$ denotes the price elasticity of demand. The demand-scaling parameter fluctuates randomly in accordance with the stochastic differential equation

$$\frac{dY(t)}{Y(t)} = \alpha dt + \sigma dW(t), \quad Y(0) = y, \quad (2)$$

where $\alpha = \mu - \delta$ denotes the drift $\delta > 0$ below the required rate of return μ , $\sigma > 0$ denotes volatility, and $W(t)$ is a standard Wiener process under the probability measure \mathbf{P} .

Correspondingly, demand dynamics in a risk-neutral world are given by

$$\frac{dY(t)}{Y(t)} = \hat{\alpha} dt + \sigma d\hat{W}(t), \quad Y(0) = y, \quad (3)$$

where $\hat{W}(t)$ is a standard Wiener process under the equivalent martingale measure $\hat{\mathbf{P}}$.

A straightforward way to determine μ , the rate of return shortfall δ , and also the risk-neutral drift $\hat{\alpha}$ would be to simply apply the CAPM, according to which

$$\mu - r = \phi \rho \sigma, \quad (4)$$

where $\phi \equiv (r_m - r)/\sigma_m$ is the market price of volatility risk and ρ denotes the correlation between returns on a hypothetical twin security and the return on the market portfolio.¹ Clearly, the market price of risk in a risk-neutral world is zero ($\phi = 0$), which implies a risk-neutral drift of $\hat{\alpha} = r - \delta$. Since $\beta \equiv \rho\sigma/\sigma_m$, the same result can be obtained via $\hat{\alpha} = r - (\mu - \alpha) = \alpha - \beta(r_m - r)$.

¹ The twin security (or portfolio) is perfectly correlated with the underlying state variable.

3.2 Monopoly

At any instant the incumbent local exchange carrier (ILEC) offers the quantity $Q(t) = Q^{\text{ILEC,m}}(t) \geq 0$ and makes a profit of

$$\begin{aligned} \Pi^{\text{ILEC,m}}(Y(t); u^{\text{ILEC,m}}(t)) \equiv & \left(P(t) - (c + c^{\text{ILEC}}) \right) Q^{\text{ILEC,m}}(t) \\ & - (C + C^{\text{ILEC}}), \end{aligned} \quad (5)$$

where c and c^{ILEC} denote variable costs, while C and C^{ILEC} denote fixed costs, attributable to the provision of basic services and distribution, respectively.

Figure 1, for the sake of completeness, illustrates the model setup for the monopolistic benchmark case, to be extended by the models that follow.

[Insert figure 1 about here.]

The payoff over the whole period $[t, T]$ is

$$J^{\text{ILEC,m}}(y, 0; u^{\text{ILEC,m}}(\cdot)) \equiv \mathbf{E}_{\mathbf{P}} \left[\int_0^T e^{-rs} \Pi^{\text{ILEC,m}}(Y(s); u^{\text{ILEC,m}}(s)) ds \right], \quad (6)$$

where $u^{\text{ILEC,m}}(\cdot) = \{u^{\text{ILEC,m}}(s), s \in [t, T]\}$ is a control process with $u^{\text{ILEC,m}}(t) \equiv Q^{\text{ILEC,m}}(t)$ and $\mathbf{E}_{\mathbf{P}}[\cdot]$ denotes expectation under the risk-neutral measure. Of course, the monopolist maximizes present value:

$$V^{\text{ILEC,m}}(Y(t), t) \equiv \sup_{u^{\text{ILEC,m}}(\cdot)} J^{\text{ILEC,m}}(Y(t), t; u^{\text{ILEC,m}}(\cdot)). \quad (7)$$

In the absence of adjustment costs, the ILEC has no incentive to deviate from the instantaneous optimum.

Proposition 1. *The monopolist chooses*

$$(Q^{\text{ILEC,m}})^*(t) = \left(\frac{Y(t)}{P^*(t)} \right)^{1/\eta} \quad (8)$$

with

$$P^*(t) = \frac{c + c^{\text{ILEC}}}{1 - \eta} \geq 0, \quad (9)$$

leading to a maximum instantaneous profit rate of

$$\left(\Pi^{\text{ILEC,m}}\right)^*(t) = \eta \left(\frac{c + c^{\text{ILEC}}}{1 - \eta}\right)^{1-1/\eta} \left(Y(t)\right)^{1/\eta} - (C + C^{\text{ILEC}}). \quad (10)$$

Proof of Proposition 1. Solving

$$(1 - \eta) \left(\frac{1}{(Q^{\text{ILEC}})^*(t)}\right)^\eta - (c + c^{\text{ILEC}}) = 0 \quad (11)$$

for the optimal quantity yields a solution candidate. It corresponds to a maximum, because

$$-(1 - \eta) \eta Y(t) \left(\frac{c + c^{\text{ILEC}}}{1 - \eta}\right)^{1+1/\eta} < 0 \quad (12)$$

if demand is price-elastic ($\eta < 1 \Leftrightarrow \varepsilon > 1$). \square

For simplicity, the following analysis is thus limited to the convenient case of price-elastic demand. In addition, as shown below, the present value of profits is bounded if and only if $\eta > 1 - \delta/r$.

Proposition 2. *Assuming $\eta \in (1 - \delta/r, 1)$ the monopolist's business is worth*

$$\begin{aligned} V^{\text{ILEC,m}}(y, 0) = & \left(1 - e^{-(r-(r-\delta)/\eta)T}\right) \\ & \cdot \eta \left(\frac{c + c^{\text{ILEC}}}{1 - \eta}\right)^{1-1/\eta} y^{1/\eta} \frac{1}{r - (r - \delta)/\eta} \\ & - (1 - e^{-rT}) (C + C^{\text{ILEC}}) \frac{1}{r}. \end{aligned} \quad (13)$$

Proof of Proposition 2. Given that

$$\mathbf{E}_{\mathbf{P}}[Y(t)] = e^{(r-\delta)t} y, \quad (14)$$

we have

$$V^{\text{ILEC},m}(y, 0) = \int_0^T e^{-rs} \left(\eta \left(\frac{c + c^{\text{ILEC}}}{1 - \eta} \right)^{1-1/\eta} \left(e^{(r-\delta)s} y \right)^{1/\eta} - (C + C^{\text{ILEC}}) \right) ds. \quad (15)$$

Straightforward integration verifies the proposition. \square

The first addend in (13) is just the present value of future contribution margins, the second addend the present value of future fixed costs. Note that, under the risk-neutral measure, the contribution margin effectively grows at a rate $(r - \delta)/\eta$, which is an increasing function of price elasticity. More formally, this result follows from applying Itô's Lemma to (10).

3.3 Duopoly

A more complex decision problem is depicted in figure 2. The incumbent acts as a monopolist, until a competitive local exchange carrier (CLEC) enters at time τ^{CLEC} and both carriers form a regulated duopoly.

[Insert figure 2 about here.]

Total quantity becomes $Q(t) = Q^{\text{ILEC},d}(t) + Q^{\text{CLEC}}(t)$. The incumbent's payoff is

$$J^{\text{ILEC}}(y, 0; u^{\text{ILEC}}(\cdot)) \equiv \mathbf{E}_{\mathbf{P}} \left[\int_0^{\tau^{\text{CLEC}}} e^{-rs} \Pi^{\text{ILEC},m}(Y(s); u^{\text{ILEC},m}(s)) ds + \int_{\tau^{\text{CLEC}}}^T e^{-rs} \Pi^{\text{ILEC},d}(Y(s); u^{\text{ILEC},d}(s)) ds \right]. \quad (16)$$

Once the CLEC has entered, the incumbent earns a profit flow of

$$\begin{aligned} \Pi^{\text{ILEC},d}(Y(t); u^{\text{ILEC},d}(t)) &\equiv \left(P(t) - (c + c^{\text{ILEC}}) \right) Q^{\text{ILEC},d}(t) \\ &+ \left(P^{\text{CLEC}}(t) - c \right) Q^{\text{CLEC}}(t) - (C + C^{\text{ILEC}}), \end{aligned} \quad (17)$$

where $Q^{\text{CLEC}}(t) \geq 0$ is the entrant's quantity and $P^{\text{CLEC}}(t)$ is the access price charged by the ILEC. Consequently, the control is vector-valued for all $t \in [\tau^{\text{CLEC}}, T]$, that is

$$u^{\text{ILEC}}(t) \equiv \begin{cases} Q^{\text{ILEC,m}}(t) & \text{if } t < \tau^{\text{CLEC}}, \\ (Q^{\text{ILEC,d}}(t), P^{\text{CLEC}}(t)) & \text{otherwise.} \end{cases} \quad (18)$$

The incumbent's business is worth

$$V^{\text{ILEC}}(y, 0) \equiv \sup_{u^{\text{ILEC}}(\cdot)} J^{\text{ILEC}}(y, 0; u^{\text{ILEC}}(\cdot)). \quad (19)$$

Due to regulatory constraints, the access price may not exceed a pre-specified cap ($P^{\text{CLEC}}(t) \leq P^{\text{CLEC,max}}(t)$). More specifically, the maximum access price corresponds to the ILEC's average total costs:

$$P^{\text{CLEC,max}}(t) \equiv c + \frac{Q^{\text{CLEC}}(t)}{Q(t)}C. \quad (20)$$

Furthermore, the CLEC should not be forced to accept margins resulting in losses (*margin squeeze*), implying a floor on the consumer price ($P(t) \geq P^{\text{min}}(t)$):

$$P^{\text{min}}(t) \equiv P^{\text{CLEC}}(t) + c^{\text{CLEC}} + \frac{C^{\text{CLEC}}}{Q^{\text{CLEC}}(t)}. \quad (21a)$$

For reasons of practicality, it is sometimes preferable to set

$$P^{\text{min}}(t) \equiv (1 + \psi) P^{\text{CLEC,max}}(t), \quad (21b)$$

where $\psi > 0$ is some positive constant, usually around 25%.

Regardless of the specific rule in place, the entrant's payoff becomes

$$J^{\text{CLEC}}(Y(t), t; Q^{\text{CLEC}}(\cdot)) \equiv \mathbf{E}_{\hat{\mathbf{P}}} \left[\int_t^T e^{-r(s-t)} \cdot \Pi^{\text{CLEC}}(Y(s); Q^{\text{CLEC}}(s)) \, ds \right]. \quad (22)$$

At time $t = 0$ the value of the CLEC's business $F^{\text{CLEC}}(0)$ thus depends on the control process $u^{\text{CLEC}}(\cdot) \equiv Q^{\text{CLEC}}(\cdot)$ and the time of market entry τ^{CLEC} , that is

$$V^{\text{CLEC}}(Y(t), t) \equiv \sup_{u^{\text{CLEC}}(\cdot)} J^{\text{CLEC}}(Y(t), t; u^{\text{CLEC}}(\cdot)) \quad (23)$$

and

$$F^{\text{CLEC}}(y, 0) \equiv \sup_{\tau^{\text{CLEC}}} \mathbf{E}_{\mathbf{P}} \left[e^{-r\tau^{\text{CLEC}}} \cdot \max \left\{ V^{\text{CLEC}}(Y(\tau^{\text{CLEC}}), \tau^{\text{CLEC}}) - I^{\text{CLEC}}, 0 \right\} \right], \quad (24)$$

where I^{CLEC} is the entrant's initial investment.

Knowing the optimal quantities and prices, the incumbent could still offer higher quantities and increase access prices to put pressure on the entrant. However, since the entrant, by assumption, has no other option but to fight, such an aggressive strategy is based on an *incredible threat*, which implies that the ILEC should accommodate. This said, the problem simplifies considerably, because the incumbent will always pick the myopically optimal control and maximize the instantaneous profit rate.² It is up to the CLEC to pick an optimal time for market entry.

As it can be difficult to find analytical solutions to the type of constrained maximization problem described above, the reaction functions required to obtain equilibrium strategies are not readily available. For each player, we therefore employ numerical techniques to derive optimal controls contingent on the other player's strategy. We then use a fixed point search, repeatedly choosing best responses. The algorithm converges to an evolutionary stable Cournot–Nash equilibrium.³ Convergence of the algorithm is shown graphically in figure 3.

² For a discussion of myopically optimal strategies in option games consult Pawlina and Kort (2006) and the references therein.

³ A more detailed account of the theoretical underpinnings of such procedures is provided by Becker and Chakrabarti (2005).

[Insert figure 3 about here.]

Figure 4 illustrates how equilibrium quantities and prices change as a function of demand.⁴ For low levels of demand, the entrant is able to capture significant market share. The access price charged by the ILEC stays well below the permitted maximum. With rising demand, the incumbent increases both quantity and access price. The latter eventually reaches the regulatory threshold, while quantities continue to grow. Needless to say, price regulation leads to a sizeable wealth transfer from ILEC to CLEC.

[Insert figure 4 about here.]

Of course, if, contrary to the example depicted in figure 3, the cost structure is asymmetric, equilibrium quantities and prices differ substantially.

Finally, in order to assess value implications, we construct a binomial tree approximating the continuous-time dynamics of demand (Cox et al., 1979).⁵

As usual, we set

$$u \equiv 1/d = e^{\sigma\sqrt{\Delta t}} \quad (25)$$

and

$$p = \frac{e^{(r-\delta)\Delta t} - d}{u - d}, \quad (26)$$

which is the risk-neutral probability of an upward movement, accounting for the fact that the underlying asset may earn a below-equilibrium rate of return (McDonald and Siegel, 1984).

Let $i \in \{1, \dots, n+1\}$ with $n = T/\Delta t$ denote the current period and $j-1$ the number of downward jumps. Valuation proceeds backward in time, starting

⁴ Note that apparent discontinuities in the graphs are merely due to the numerical method employed.

⁵ Alternatively, the log-transformed model described by Trigeorgis (1991) yields more accurate and robust results.

with the CLEC's entry decision. To this purpose, a tree containing the payoffs from immediate entry is constructed. At the leaves we have

$$V_{j,n+1}^{\text{CLEC}} = \left(\Pi_{j,n+1}^{\text{CLEC}}\right)^* \Delta t. \quad (27)$$

For all other nodes

$$V_{j,i}^{\text{CLEC}} = \left(\Pi_{j,i}^{\text{CLEC}}\right)^* \Delta t + e^{-r\Delta t} \left(p V_{j,i+1}^{\text{CLEC}} + (1-p) V_{j+1,i+1}^{\text{CLEC}} \right). \quad (28)$$

At each node, the CLEC decides whether it is advantageous to continue to wait or to incur initial costs of I^{CLEC} and enter the market:

$$F_{j,n+1}^{\text{CLEC}} = \max \left\{ V_{j,n+1}^{\text{CLEC}} - I^{\text{CLEC}}, 0 \right\} \quad (29)$$

and

$$F_{j,i}^{\text{CLEC}} = \max \left\{ V_{j,i}^{\text{CLEC}} - I^{\text{CLEC}}, e^{-r\Delta t} \left(p F_{j,i+1}^{\text{CLEC}} + (1-p) F_{j+1,i+1}^{\text{CLEC}} \right) \right\}. \quad (30)$$

The optimal policy derived can then be used to determine the value of the ILEC's business. If the CLEC is active we set

$$V_{j,i}^{\text{ILEC}} = \left(\Pi_{j,i}^{\text{ILEC,d}}\right)^* \Delta t + e^{-r\Delta t} \left(p V_{j,i+1}^{\text{ILEC}} + (1-p) V_{j+1,i+1}^{\text{ILEC}} \right) \quad (31)$$

and

$$V_{j,i}^{\text{ILEC}} = \left(\Pi_{j,i}^{\text{ILEC,m}}\right)^* \Delta t + e^{-r\Delta t} \left(p V_{j,i+1}^{\text{ILEC}} + (1-p) V_{j+1,i+1}^{\text{ILEC}} \right) \quad (32)$$

otherwise.

Consider an illustrative example with $y = 30$, $r = 0.05$, $r_m = 0.15$, $\beta = 1.5$, $\alpha = 0.15$, $\sigma = 0.3$, $T = 1$, $n = 4$, $\eta = 0.5$, $c = 5$, $c^{\text{ILEC}} = 1$, $c^{\text{CLEC}} = 1$, $C = 10$, $C^{\text{ILEC}} = 5$, $C^{\text{CLEC}} = 5$, and $I^{\text{CLEC}} = 0.5$. Table 1 shows the demand-scaling parameter ($Y_{j,i}$), table 2 the corresponding equilibrium quantities for the ILEC and the CLEC ($Q_{j,i}^{\text{ILEC,d}}$, $Q_{j,i}^{\text{CLEC}}$) as well as the access price ($P_{j,i}^{\text{CLEC}}$). The data essentially verify conclusions drawn from figure 4.

[Insert table 1 about here.]

[Insert table 2 about here.]

Table 3 lists profit flows for both competitors ($\Pi_{j,i}^{\text{ILEC,d}}$, $\Pi_{j,i}^{\text{CLEC}}$, $\Pi_{j,i}^{\text{ILEC,m}}$). For low levels of demand, the incumbent incurs losses, while the entrant benefits from price regulation and is able to limit downside risk.

[Insert table 3 about here.]

Downside protection also drives the results of table 4, which provides information on the value of the option wait ($F_{j,i}^{\text{CLEC}}$), and the corresponding optimal policy. In addition, the table lists the value of the ILEC's business ($V_{j,i}^{\text{ILEC}}$) at each node.

[Insert table 4 about here.]

Based on the replicating portfolio approach, it is also possible to determine the systematic risk inherent in the claim and thus the appropriate cost of equity capital. The portfolio consists of

$$m_{j,i} = \frac{e^{\delta\Delta t} (F_{j,i+1}^{\text{ILEC}} - F_{j+1,i+1}^{\text{ILEC}})}{(u-d)Y_{j,i}} \quad (33)$$

units of the twin security and an investment in the risk-free asset of

$$B_{j,i} = \frac{uF_{j+1,i+1}^{\text{ILEC}} - dF_{j,i+1}}{(u-d)e^{r\Delta t}}. \quad (34)$$

Since, by definition, the risk-free asset carries zero systematic risk ($\beta^B \equiv 0$), the systematic risk of the replicating portfolio, corresponding to the systematic risk of the contingent claim to be valued, is

$$\beta_{j,i}^F = \frac{mY_{j,i}}{F_{j,i}^{\text{ILEC}}}\beta. \quad (35)$$

While immediately apparent to everyone familiar with option pricing, it is important to point out the fact that no single risk-adjusted discount rate properly and fully captures the risk or potential value stemming from demand uncertainty. The cost of capital is clearly state-dependent and, as such, fluctu-

ates with demand. Generally speaking, price regulation of the form described above tends to increase the cost of capital and thus decrease the value of the ILEC's business.

If the ILEC is not active from the outset and, similar to the CLEC, holds an *option to wait*, the same logic as before applies and the ILEC will accommodate (fig. 5). The value of the option to wait reflects the incumbent's willingness to commit to upfront investments into technology and infrastructure.

[Insert figure 5 about here.]

Based on the previously constructed tree describing $V_{j,i}^{\text{ILEC}}$, the value of the option to wait $F_{j,i}^{\text{ILEC}}$ with an initial investment of I^{ILEC} is easily derived.

4 Variations and Extensions

Once the CLEC holds an *option to abandon*, the ILEC potentially has an incentive to increase access prices and quantities, with the goal of inducing a margin squeeze and forcing the CLEC to exit the market. The timeline for the new decision problem is shown in figure 6.

[Insert figure 6 about here.]

For computational reasons it seems advisable to assume that the ILEC will either fight or accommodate the new entrant, but, regardless of changes in demand, will not revise the strategy, once chosen. The problem thereby becomes tractable with basic methods that would otherwise suffer the *curse of dimensionality*, calling for more sophisticated techniques such as Kushner's Markov chain approximation for non-zero-sum stochastic differential games (Kushner, 2007; Kushner and Dupuis, 2001).

Analysis of the extended problem follows the familiar steps from previous sections. Based on a binomial model, it is also possible to derive alternative price paths and draw conclusions as to value-maximizing strategies for the incumbent and the new entrant.

5 Conclusion

Flexibility drives value. However, as was demonstrated in detail, the lack of flexibility resulting from regulatory intervention, namely the inability to freely choose quantities and prices in response to changes in demand, destroys option value originally inherent in the ILEC's business. This view is complementary to the one adopted in other contributions, which explicitly stress the value created for a more flexible entrant. The entrant is, at least partially, protected against downside risk, whereas the incumbent bears the burden of substantial fixed costs and experiences substantial increases in the cost of capital. Further research is required to explore in more detail the implications of (in-)flexibility brought about by price controls under a variety of assumptions. In particular, extending the analysis to more complex optimal control problems seems promising. Findings should be of immediate relevance to telecommunications and other network industries, for example including energy.

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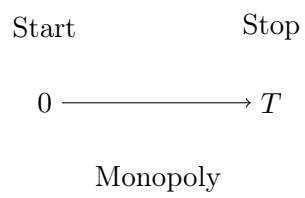


Fig. 1. Timeline (model 1)

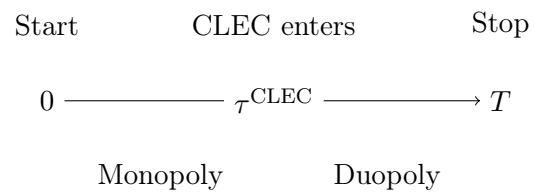


Fig. 2. Timeline for market entry (model 2)

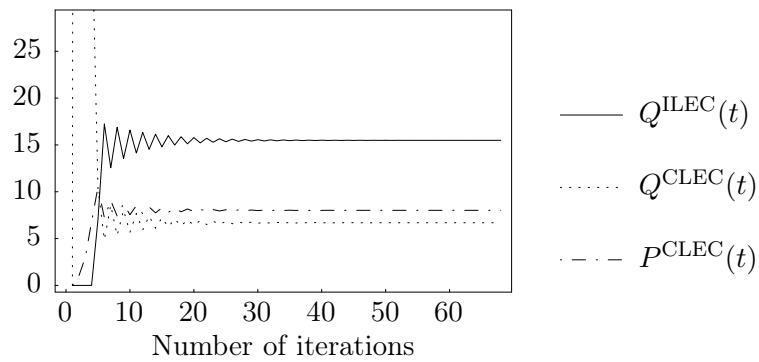


Fig. 3. Convergence of the fixed point search employed to identify Cournot–Nash equilibria ($y = 50$, $\eta = 0.5$, $c = 5$, $c^{\text{ILEC}} = 1$, $c^{\text{CLEC}} = 1$, $C = 10$, $C^{\text{ILEC}} = 5$, and $C^{\text{CLEC}} = 5$).

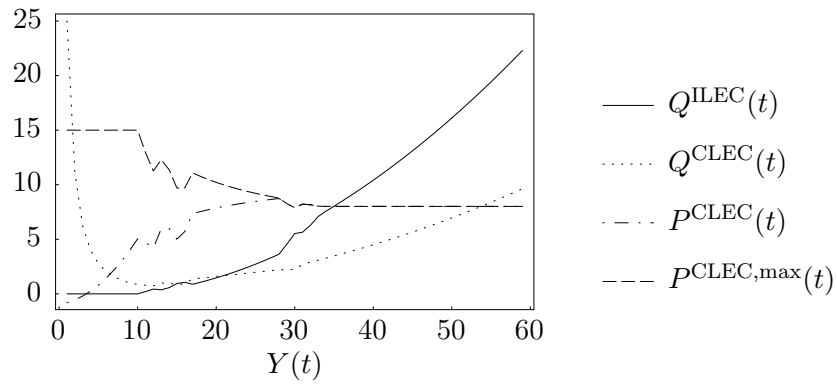


Fig. 4. Equilibrium quantities and prices as a function of demand ($\eta = 0.5$, $c = 5$, $c^{\text{ILEC}} = 1$, $c^{\text{CLEC}} = 1$, $C = 10$, $C^{\text{ILEC}} = 5$, and $C^{\text{CLEC}} = 5$).

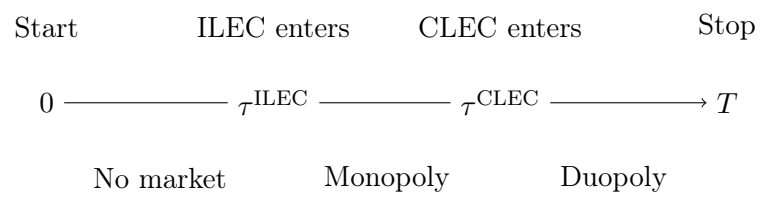


Fig. 5. Timeline for sequential market entry (model 3)

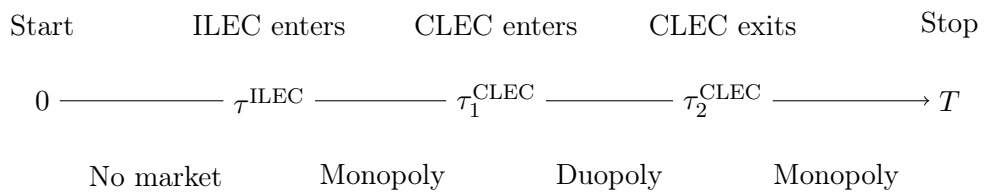


Fig. 6. Timeline for sequential market entry and exit (model 4)

Table 1
Demand-scaling parameter.

j	Demand-scaling parameter				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
1	30.0000	34.855	40.4958	47.0494	54.6636
2		25.8212	30.0000	34.855	40.4958
3			22.2245	25.8212	30.0000
4				19.1288	22.2245
5					16.4643

Table 2
Quantities for the ILEC and the CLEC, access price.

j	Equilibrium values				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
1	3.7279	7.5266	10.1600	13.7146	18.5136
	2.2118	3.2503	4.3875	5.9226	7.9953
	9.0175	8.0163	8.0162	8.0162	8.0159
2		2.6660	3.7279	7.5266	10.1600
		1.9644	2.2118	3.2503	4.3875
		8.4544	9.0175	8.0163	8.0162
3			1.7393	2.6660	3.7279
			1.6906	1.9644	2.2118
			8.0428	8.4544	9.0175
4				1.0860	1.7393
				1.4553	1.6906
				7.5637	8.0428
5					0.0000
					0.9519
					7.4378

Table 3
 Duopoly profit flows for the ILEC and CLEC, monopoly profit flow for the ILEC.

j	Profit flows for both competitors				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
1	17.4070	29.5572	45.1458	66.1881	94.5907
	0.0693	0.2041	2.0248	4.4827	7.8011
	22.5000	35.6197	53.3295	77.2351	109.5044
2		7.7807	17.4070	29.5572	45.1458
		0.0000	0.0693	0.2041	2.0248
		12.7807	22.5000	35.6197	53.3295
3			0.5804	7.7807	17.4070
			0.0000	0.0000	0.0693
			5.5804	12.7807	22.5000
4				-4.7536	0.5804
				0.0000	0.0693
				0.2464	5.5804
5					-12.6796
					3.0315
					3.7052

Table 4

Value of the option to wait, optimal policy, value of the ILEC's business.

j	Option to wait, policy, ILEC's business value				
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
1	0.2721 Wait 5.4279	0.5501 Wait 8.3849	1.2026 Enter 11.9926	1.7803 Enter 16.7932	1.4503 Enter 23.6477
2		0.0391 Wait 3.0097	0.0013 Wait 5.4760	0.0028 Wait 8.1414	0.0062 Enter 11.2865
3			0.0726 Wait 0.9574	0.0000 Wait 3.3101	0.0000 Wait 5.6250
4				0.1369 Wait -1.0451	0.0000 Wait 1.3951
5					0.2579 Enter -3.1699