Coalition Formation under Uncertainty: the Case of First-mover Disadvantage (Preliminary Version)

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We discuss the problem of value creation via mergers in a dynamic framework of Bertrand competition under uncertainty for the case of an oligopoly consisting of three (and N) firms. To do so we extent the framework of the Deneckere and Davidson (1985) model into a stochastic dynamic set-up.

Three factors that affect timing of decisions are discussed: the form of market uncertainty (whether market is growing or declining), the impact of market microstructure (the substitutability parameter among brands offered by firms), and the size of the merger $M \leq N$.

1 The mathematical framework of the model

Let time be continuous and indexed by $t \geq 0$. Consider a horizontally differentiated oligopoly with three firms named $1, 2$ and 3. Each firm is described by its brand demand function and for simplicity there are no costs of production. The firm $i \in \{1, 2, 3\}$ produces one brand and charges at time t the unit price p_{it} for the good of the own

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brand. Demand is specified in the manner of Shubik (1980):

$$q_{it}(p_{1t}, p_{2t}, p_{3t}, Y_t) = \sqrt{Y_t} - p_{it} - \gamma\Big(p_{it} - \frac{1}{3}\sum_{j=1}^{3} p_{jt}\Big),$$

where q_{it} is the quantity demanded of firm i's brand at time t, γ is a substitutability parameter, and $(\sqrt{Y_t})_{t\geq 0}$ describes the evolution of consumers' willingness to buy. The stochastic intercept of the demand function, $(\sqrt{Y_t})_{t\geq 0}$, captures aggregate uncertainty in the economics and is represented by a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{Q})$. The profit of the i-th firm is described as

$$\pi_{it}(p_{1t}, p_{2t}, p_{3t}, Y_t) = p_{it} q_{it}(p_{1t}, p_{2t}, p_{3t}, Y_t),$$

and it is assumed that price is a strategic variable of the firm (Bertrand competition). At the initial point in time all firms are active and the market is in equilibrium. In equilibrium every firm charges price $p_t = \frac{\sqrt{Y_t}}{2(1+\frac{\gamma}{3})}$. This implies that the profit of i-th firm is equal to

$$\pi_t = p_t q_t = \frac{(1+\gamma\frac{2}{3})}{(2+\gamma\frac{2}{3})^2} Y_t \quad \text{for } i = 1, 2, 3.$$

Focus now on the situation, in which two firms consider merging together at time t and charging identical prices from this moment in time on, while the remaining firm continues to act independently and reacts to the price change of its opponents. The problem is to determine the optimal timing of the coalition formation decision and to derive the corresponding price configuration. Let us indicate the main results related to price setting game first.

Let p_{Lt} denote the new unit price charged by coalition members (for each brand they own) and let p_{Ft} be the price charged by the outsider. As shown in the Appendix, the equilibrium prices are:

$$p_{Lt} = \frac{(6+5\gamma)}{12+12\gamma+2\gamma^2}\sqrt{Y_t} \quad \text{and} \quad p_{Ft} = \frac{(6+4\gamma)}{12+12\gamma+2\gamma^2}\sqrt{Y_t}$$

so $p_{Lt} > p_{Ft} > p_t$ $\mathcal{Q}$-a.s. for $\gamma > 0$ and the corresponding profits are as follows

$$\begin{aligned}
\pi_{Lt} &= p_{Lt}^2\Big(1+\frac{\gamma}{3}\Big) = \frac{(6+5\gamma)^2}{(12+12\gamma+2\gamma^2)^2}\Big(1+\frac{\gamma}{3}\Big)Y_t \\
\pi_{Ft} &= p_{Ft}^2\Big(1+\frac{2\gamma}{3}\Big) = \frac{(6+4\gamma)^2}{(12+12\gamma+2\gamma^2)^2}\Big(1+\frac{2\gamma}{3}\Big)Y_t.
\end{aligned}$$

Alternatively, suppose that three firms consider merging to a monopoly and charging the identical price. The equilibrium price is then $p_{Jt} = \frac{1}{2}\sqrt{Y_t}$ and the corresponding profit is $\pi_{Jt} = \frac{1}{4}Y_t$.

In order to simplify presentation we adopt the following notation:

Definition 1.1 *Let*

$$\begin{aligned}
D &:= \frac{(1+\gamma\frac{2}{3})}{(2+\gamma\frac{2}{3})^2} \\
D_L &:= \frac{(6+5\gamma)^2}{(12+12\gamma+2\gamma^2)^2}\Big(1+\frac{\gamma}{3}\Big) \\
D_F &:= \frac{(6+4\gamma)^2}{(12+12\gamma+2\gamma^2)^2}\Big(1+\frac{2\gamma}{3}\Big) \\
D_J &:= \frac{1}{4}.
\end{aligned}$$

By straightforward algebra we obtain the following result about the profit flows before and after the merger takes place.

Remark 1.1 *For every $\gamma > 0$ it holds that $0 < D < D_L < D_F < D_J$, what implies that $\pi_t < \pi_{Lt} < \pi_{Ft} < \pi_{Jt}$ $\mathcal{Q}$-a.s.*

The focus of our analysis is on the optimal timing of the merger. We start out by solving the case, in which the merger resulting in a monopoly is not allowed by the competition authority and we solve the game with exogenous role assignment of the firms.

To push our analysis further we specify the uncertainty. The shock process $(Y_t)_{t\geq 0}$ follows a geometric Brownian motion under $\mathcal{Q}$:

$$dY_t = Y_t(\mu dt + \sigma dB_t),$$

where μ is a constant denoting drift, and σ is a positive constant related to the instantaneous standard deviation, and dB_t is the increment of a Wiener process. The process $(Y_t)_{t\geq 0}$ starts at $Y_0 > 0$ $\mathcal{Q}$-a.s.

To make the economic interpretation easier we derive all results in terms of pre-merger profit, which under $\mathcal{Q}$ follows geometric Brownian motion

$$d\pi_t = \pi_t(\mu dt + \sigma dB_t),$$

where μ, σ and dB_t are as before and $\mathcal{Q}(\pi_0 = DY_0) = 1$. This results in the following description of the post-merger profit rates in terms of pre-merger profit rate:

$$\pi_{Lt} = \frac{D_L}{D}\pi_t \quad \text{and} \quad \pi_{Ft} = \frac{D_F}{D}\pi_t.$$

Throughout the paper we use the deterministic discount factor $dR_t = -R_t r dt$ with $R(0) = 1$ and $r > \mu$.

2 The basic model without strategic effect

Decisions about undertaking the merger activity are solved by application of real options methodology. To start out we define the value of each firm in all situations that may arise due to the merger activity and we determine the optimal value maximizing policy for the firms. For the time being, let us assume that *one firm is passive* (say firm 3) in the sense that it never proposes coalition formation to any firm and it will never be involved in a merger. For this case, the model can be solved as if the roles of the firms were assigned and the equilibrium price configurations are taken as given.

The value of the firm at time t is described by the value of initial assets in place and the value of the firm's growth options. The value of the assets in place at time t is modelled as follows

$$A(\pi_t) = \mathbb{E}_t \int_t^\infty \frac{R_s}{R_t}\pi_s ds = \frac{\pi_t}{r - \mu}.$$

We focus on the situation where two firms have an opportunity to create a coalition with an other firm (e.g. collaboration on R&D or M&A), but we exclude the possibility of the creation of a big coalition of three firms.

Suppose that two firms (firm 1 and 2) consider the possibility of merging together. The sunk cost of the merger is $K > 0$ per firm involved in the merger, so it is assumed that the merger costs incurred are equal among the firms participating in merger. The value at time t of a brand owned by the coalition if the merger takes place at stochastic time T_L is defined as follows:

$$L(\pi_t) = \mathbb{E}_t \int_t^\infty \frac{R_s}{R_t}\pi_s ds + \mathbb{E}_t\left(\int_{T_L}^\infty \frac{R_s}{R_t}\frac{D_L - D}{D}\pi_s ds - \frac{R_T}{R_t}K\right) \geq A_t(\pi_t),$$

in which $\frac{D_L - D}{D} > 0$ denotes the return on merger, so $\frac{D_L - D}{D}\pi_s > 0$ captures the increase in profit due to the merger, $T_L = \inf(t \geq 0 | \pi_t \geq \pi^*)$, where π^* is the threshold profit level for the merger to take place.

After the merger comes about, the market changes from three to two firms. The implication is that competition declines so that also the value of the firm remaining outside the alliance changes (see Proposition 1.1). The value of the outsider implied by the optimal adjustment of the outsider's price to post-merger prices is:

$$F(\pi_t) = \mathbb{E}_t \int_t^{T_L} \frac{R_s}{R_t}\pi_s ds + \mathbb{E}_t \int_{T_L}^{\infty} \frac{R_s}{R_t}\frac{D_F}{D}\pi_s ds.$$

Each firm holding the merger option maximizes its own value:

$$L(\pi_t) = \max_{T \geq t} \left\{ \mathbb{E}_t \int_t^{\infty} \frac{R_s}{R_t}\pi_s ds + \mathbb{E}_t \left(\int_T^{\infty} \frac{R_s}{R_t}\frac{D_L - D}{D}\pi_s ds - \frac{R_T}{R_t}K \right) \right\}$$

Because of the sunk cost associated with the merger the firms find it optimal to exercise their merger option when the profit rate is sufficiently large. The stopping region for the pre-merger profit level π_t can be expressed as a threshold level π^* such that it is optimal to exercise the merger option when π_t exceeds π^*. It is a standard result (see, e.g. Dixit and Pindyck (1996)) that as long as it is not optimal to merge, the value function $L(\cdot)$ must satisfy the following differential equation, which results from Bellman's principle of optimality and the application of Itô's lemma:

$$\frac{\sigma^2}{2}\pi^2 L_{\pi,\pi}(\pi) + \mu\pi L_\pi(\pi) - rL(\pi) + \pi = 0. \tag{1}$$

The optimal stopping problem (1) can now be solved after imposing the appropriate boundary conditions. These are: the absorbing barrier condition at zero, and the value-matching and smooth-pasting at the threshold value:

$$L(0) = 0 \qquad L(\pi^*) = \frac{D_L}{D}\frac{\pi^*}{r-\mu} - K \qquad L_\pi(\pi^*) = \frac{D_L}{D}\frac{1}{r-\mu}.$$

Solving equation (1) while applying the conditions stated above, results in the following solution for $L(\cdot)$ and π^*:

$$L(\pi_t) = \begin{cases} \frac{\pi_t}{r-\mu} + (\frac{D_L - D}{D}\frac{\pi^*}{r-\mu} - K)(\frac{\pi_t}{\pi^*})^\beta & \text{if} \quad \pi_t \leq \pi^* \\ \frac{D_L}{D}\frac{\pi_t}{r-\mu} - K & \text{if} \quad \pi_t \geq \pi^* \end{cases}$$

$$\pi^* = \frac{\beta}{\beta - 1}\frac{D}{D_L - D}K(r-\mu)$$

where β is the positive root of the characteristic equation $\frac{\sigma^2}{2}\beta(\beta-1)+\mu\beta-r=0$.

The value of the outsider's firm thus be expressed as follows:

$$F(\pi_t) = \begin{cases} \dfrac{\pi_t}{r-\mu} + \dfrac{D_F - D}{D}\dfrac{\pi^*}{r-\mu}\left(\dfrac{\pi_t}{\pi^*}\right)^\beta & \text{if} \quad \pi_t \leq \pi^* \\ \dfrac{D_F}{D}\dfrac{\pi_t}{r-\mu} & \text{if} \quad \pi_t > \pi^*. \end{cases}$$

The findings related to this nonstrategic set-up are summarized in the propositions below.

Proposition 2.1 *The merger is profitable to each of the firm in the market:* $A(\pi_t) < L(\pi_t)$ *and* $A(\pi_t) < F(\pi_t)$.

Proposition 2.2 *Exercising the option to merge creates a free-riding problem:* $L(\pi_t) < F(\pi_t)$. *Thus the firm not being involved in the merger benefits most from the merger taking place.*

In what follows we discuss the impact of fundamental parameters of the model on the threshold value that triggers the merger.

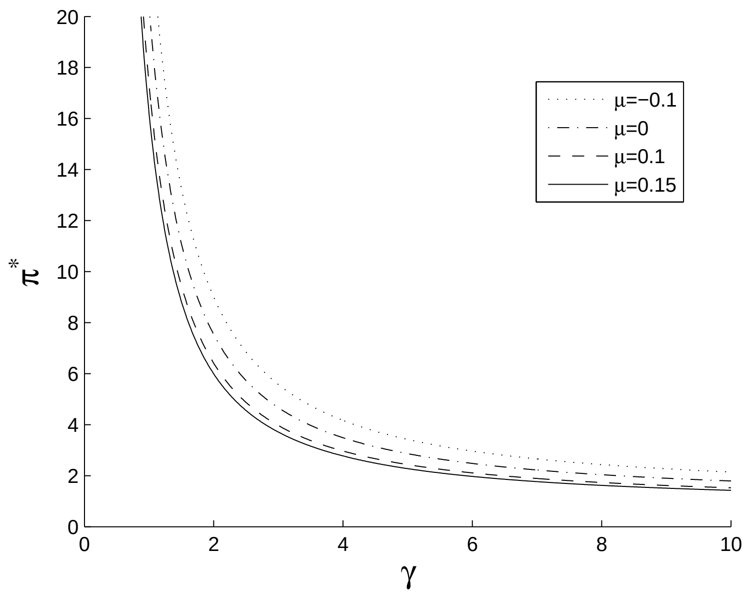

Figure 1: Impact of the substitutability parameter γ on the trigger π^*. The parameters are $(r, \sigma, K) = (0.2, 0.4, 1)$.

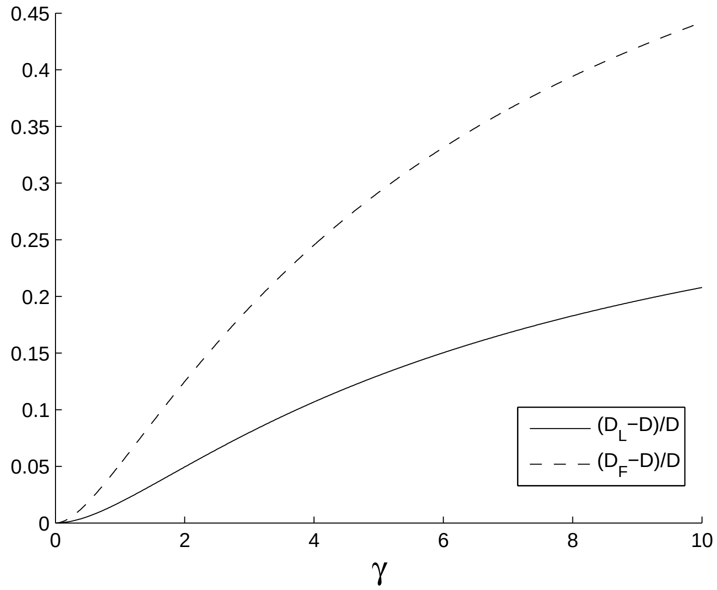

Figure 2: Impact of the substitutability parameter γ on the return from the merger for a merging party and for the outsider. The parameters are $(r, \sigma, K) = (0.2, 0.4, 1)$.

The relationship between the value of the trigger and substitutability parameter γ is depicted in Figure 1. First we notice that the value of the trigger is inversely related to the return on the merger, $\frac{D_L - D}{D}$ and the following relationship holds

$$\frac{\partial \pi^*}{\partial \gamma} = -\pi^* \frac{\partial \ln\left(\frac{D_L - D}{D}\right)}{\partial \gamma} = -\frac{\pi^*}{\frac{D_L - D}{D}} \frac{\partial \frac{D_L - D}{D}}{\partial \gamma} < 0$$

because the increase in the degree of substitutability among brands, γ results in increase of the return on the merger due to positive synergies created by merger to merging parties (as depicted in Figure 2). This delivers the incentives to create alliance what corresponds with lower value of the trigger. This result is in contrast to the prediction of the model of mergers under uncertainty and Bertrand competition proposed by Bernile et. al (2007). In their set-up the additional negative strategic effect brought by competition (the presence of the possible entrant) results in negative effect of substitutability parameter on the merger trigger.

While looking on the impact of instantaneous drift on the investment trigger we see that the increase in the consumers' willingness to buy lowers the trigger and consequently it affects positively the firm's decision about merging. However, the effect of the change in trend of the shock process on the value of the threshold that trigger mergers is found relatively small.

Let us now turn to discussion on the size of the benefit for the outsider from the merger. To do so we provide the comparative statics results corresponding with propositions 2.1 and 2.2.

Definition 2.1 *Let $\pi_t \leq \pi^*$. The ratio of the value of the second mover's gain relative to the value of the first movers' gain is:*

$$\Delta = \frac{F(\pi_t) - A(\pi_t)}{L(\pi_t) - A(\pi_t)} = \beta \frac{D_F - D}{D_L - D}.$$

Of our main interest is the impact of the substitutability parameter on the value of Δ. This relationship is illustrated in Figure 3.

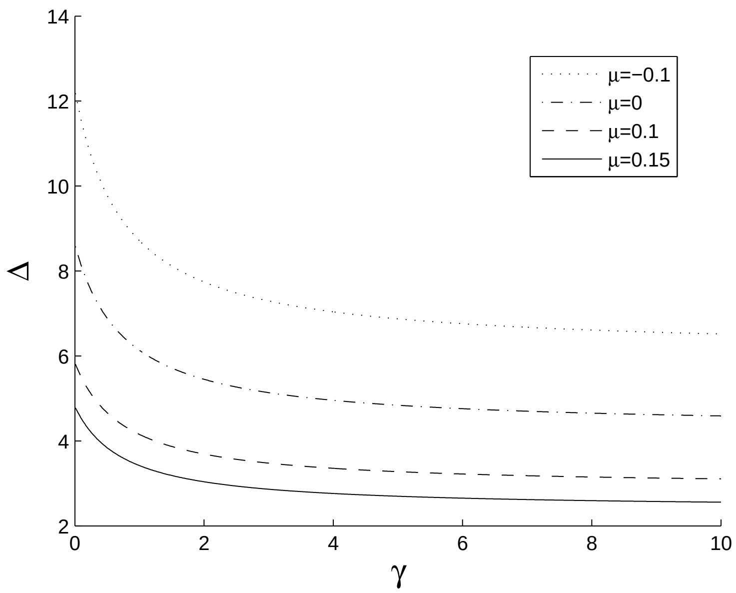

Figure 3: Impact of the substitutability parameter γ on the relative gain from the second mover advantage Δ. The parameters are $(r, \sigma, K) = (0.2, 0.4, 1)$.

It follows by easy algebra that

$$\frac{\partial\Delta}{\partial\gamma} = \Delta\left[\frac{\partial\ln\left(\frac{D_F - D}{D}\right)}{\partial\gamma} - \frac{\partial\ln\left(\frac{D_L - D}{D}\right)}{\partial\gamma}\right].$$

Thus the change in relative gain on merger for the outsider is affected only by the difference of changes in returns on mergers for the outsider and an insider. As become apparent from Figure 2, both returns are monotonically increasing in γ (the synergy effect mentioned above) and as become clear from Figure 3 the effect related to the change in the return on merger for outsider outweighs the negative effect related to the change in the return for an outsider. Additionally, we find that the level of Δ is increasing in the drift of the process describing consumers' willingness to buy.

To complete discussion let us focus on the dynamics of the mergers for the case of growing and declining industries. Following Harrison (1985), we state the probability that merger occurs in the long-run:

$$\mathcal{Q}\Big(\sup_{0\leq t<\infty} \pi_t \geq \pi^* | \pi_0 < \pi^*\Big) = \begin{cases} 1 & \text{if } \mu \geq \frac{\sigma^2}{2} \\ (\frac{\pi^*}{\pi_0})^{\frac{2\mu}{\sigma^2}-1} & \text{otherwise} \end{cases}$$

In Figure 4 the probability that merger occurs is depicted. For substantial values of the drift parameter corresponding to positive trend in the shock to the demand function merger occurs with probability 1. As the trend in consumers willingness to buy decreases the intensity of merger activity decreases. However, our model predicts that merger still takes place in industries subject to negative trend because the corresponding probability of the merger taking place in declining markets remains positive.

Remark 2.1 *The merger activity is related to the consumers' willingness to acquire goods in the market. Probability of the merger taking place increases with the increase of the trend in consumers' preferences for buying.*

3 Further discussion and extensions of the model

Suppose now that there is no restriction on the post merger market structure. Let $J(\cdot)$ denote the value of a brand owned by the grand coalition consisting of the three firms:

$$J(\pi_t) = \max_{T\geq t}\left\{\mathbb{E}_t\int_t^\infty \frac{R_s}{R_t}\pi_s ds + \mathbb{E}_t\bigg(\int_T^\infty \frac{R_s}{R_t}\frac{D_J - D}{D}\pi_s ds - \frac{R_T}{R_t}K\bigg)\right\},$$

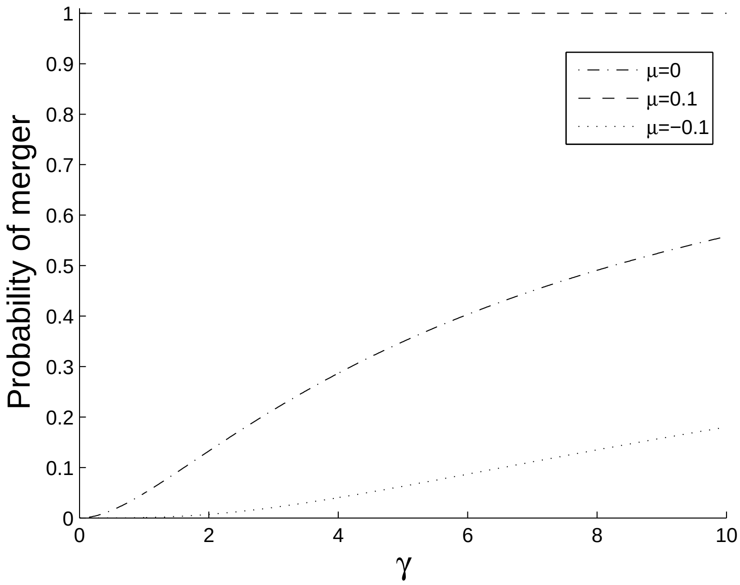

Figure 4: Impact of the substitutability parameter γ on the probability of the merger. The parameters are $(r, \sigma, K, \pi_0) = (0.2, 0.4, 1, 1)$.

where K is the sunk cost related to the merger and it is assumed that this cost is equal among firms. By standard methods one obtains that

$$J(\pi_t) = \begin{cases} \frac{\pi_t}{r-\mu} + (\frac{D_J - D}{D}\frac{\pi^{**}}{r-\mu} - K)(\frac{\pi_t}{\pi^{**}})^{\beta} & \text{if} \quad \pi_t \leq \pi^{**} \\ \frac{D_J}{D}\frac{\pi_t}{r-\mu} - K & \text{if} \quad \pi_t \geq \pi^{**} \end{cases}$$

$$\pi^{**} = \frac{\beta}{\beta - 1}\frac{D}{D_J - D}K(r-\mu) < \pi^*$$

[The version of the model with strategic interaction between two merger scenarios remains under construction]

Let us now turn to the discussion on the profitability of the mergers in oligopoly consisting of N firms. Suppose first that the merger to monopoly is not allowed and focus on the case in which there are $M < N$ firms that consider merging together.

The quantity demanded of firm i's brand is described as follows:

$$q_{it}(p_{1t}, p_{2t}, \ldots, p_{Nt}, Y_t) = \sqrt{Y_t} - p_{it} - \gamma\Big(p_{it} - \frac{1}{N}\sum_{j=1}^{N} p_{jt}\Big)$$

where p_{it} is the unit price charged by the firm i for the good of the own brand, γ is a substitutability parameter, and $(\sqrt{Y_t})_{t\geq 0}$ describes the evolution of consumers' willingness to buy, where $(Y_t)_{t\geq 0}$ follows $GBM(\mu,\sigma)$ process defined on the probabilistic space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{Q})$.

At the initial point in time the market is in equilibrium described by the N-Tuple of pre-merger equilibrium prices

$$p_{it} = \frac{1}{2+\gamma\frac{N-1}{N}}\sqrt{Y_t} \qquad \text{for } i = 1,2,\ldots,N.$$

The corresponding profits are

$$\pi_{it} = \frac{1+\gamma\frac{N-1}{N}}{(2+\gamma\frac{N-1}{N})^2}Y_t \qquad \text{for } i = 1,2,\ldots,N.$$

To simplify notation we drop index referring to firm, i, and in what follows the process $(\pi_t)_{t\geq 0}$ is named as the pre-merger profit flow process.

Focus now on the situation, in which M firms consider merging together at time t and charging identical prices from this moment in time on, while the remaining $N-M$ firms continues to operate independently. We look on the problem of the optimal timing of the coalition formation decision and we derive the price configuration corresponding to the post merger scenario. For the time being we restrict to a single merger.

Let p_{Lt} and p_{Ft} denote the price charged by the coalition members and by the outsider(s), respectively. The post merger equilibrium price configuration is the following:

$$\begin{cases} p_{Lt} = \frac{2N+\gamma(2N-1)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}\sqrt{Y_t} \\ p_{Ft} = \frac{2N+\gamma(2N-M)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}\sqrt{Y_t}, \end{cases}$$

and the corresponding to this equilibrium profit equals

$$\begin{cases} \pi_{Lt} = \left(\frac{2N+\gamma(2N-1)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}\right)^2(1+\gamma\frac{N-M}{N})Y_t \\ \pi_{Ft} = \left(\frac{2N+\gamma(2N-M)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}\right)^2(1+\gamma\frac{N-1}{N})Y_t. \end{cases}$$

Definition 3.1 *Let*

$$D(N) := \frac{1+\gamma\frac{N-1}{N}}{(2+\gamma\frac{N-1}{N})^2}$$

$$D_L(M,N) := \left(\frac{2N+\gamma(2N-1)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}\right)^2(1+\gamma\frac{N-M}{N})$$

$$D_F(N-M,N) := \left(\frac{2N+\gamma(2N-M)}{4N+2\gamma(3N-M-1)+\gamma^2\frac{N-M}{N}(2N+M-2)}V\right)^2(1+\gamma\frac{N-1}{N}).$$

Remark 3.1 *For any* $1 < M \leq N$ *and* $\gamma > 0$ *it holds that* $0 < D(N) < D_L(M, N) < D_F(N - M, N)$. *Consequently,* $\pi_t < \pi_{Lt} < \pi_{Ft}$ $\mathcal{Q}$*-a.s.*

The above remark suggests that a merger would be profitable for both the merging parties and for the outsiders if there were not sunk cost. Assume however that there is sunk cost of the merger, K, and this cost is incurred by each firms in the alliance. By analogous reasoning as for three firms we obtain the trigger at the pre-merger profit process as follows:

$$\pi^*(M, N) = \frac{\beta}{\beta - 1} \frac{D(N)}{D_L(M, N) - D(N)} K(r - \mu).$$

We observe that the value of the trigger is inversely related to the return on the merger for a merging party. Consequently, the higher the return from creating alliance the higher the incentive to merge, what correspond to lowering the merger trigger.

In order to discuss the relationship between size of the merger and state similar result as in Deneckere and Davidson (1985) (cf. Theorem 2 in their paper).

Remark 3.2 *Mergers are increasingly profitable, i.e.* $D_L(M, N) < D_L(M + 1, N) \Rightarrow \pi_{Lt}(M, N) := D_L(M, N) Y_t < D_L(M + 1, N) Y_t =: \pi_{Lt}(M + 1, N)$ $\mathcal{Q}$*-a.s.*

Consequently, because $D_L(M, N)$ is increasing in the size of the merger, and because the sunk cost of merger is independent of the merger size, it follows that the increase in the size of the merger rises the incentive to merge. The latter corresponds with lowering the trigger on pre-merger profit flow process ($\frac{\partial \pi^*(M,N)}{\partial M} < 0$). It is worth stressing here that this result may not hold true in fully strategic set-up, because then a firm may prefer free-riding to merging given the merger would come about.

Now we generalize the definition of Δ from the previous section in order to examine the impact of the size of the merger and the degree of substitution among goods on the relative gain earned by the outsider if a merger takes place.

Definition 3.2 *Let* $\pi_t \leq \pi^*(M, N)$. *The ratio of the value of the second movers's gain relative to the value of the first movers' gain is:*

$$\Delta(M, N) = \beta \frac{D_F(N - M, N) - D(N)}{D_L(M, N) - D(N)}.$$

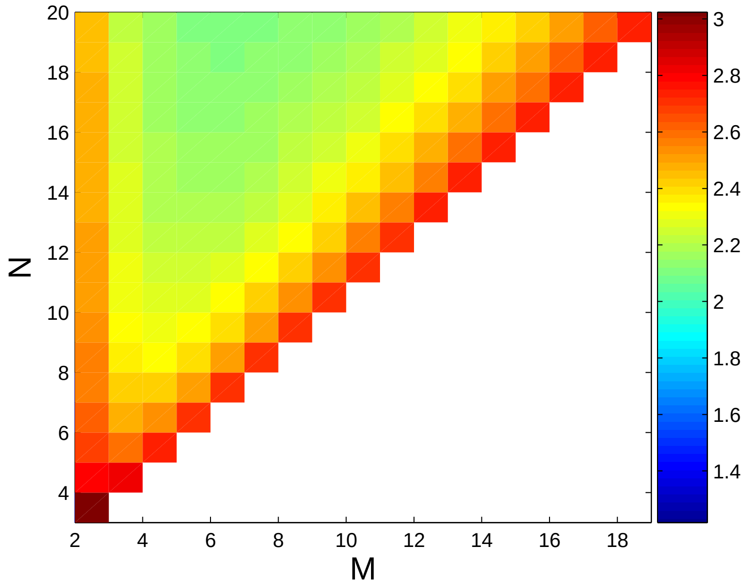

Figure 5: Impact of the size of the merger on the relative gain from the second mover advantage $\Delta(M, N)$. The parameters are $(r, \mu, \sigma, K) = (0.2, 0.15, 0.4, 1)$ and $\gamma = 2$.

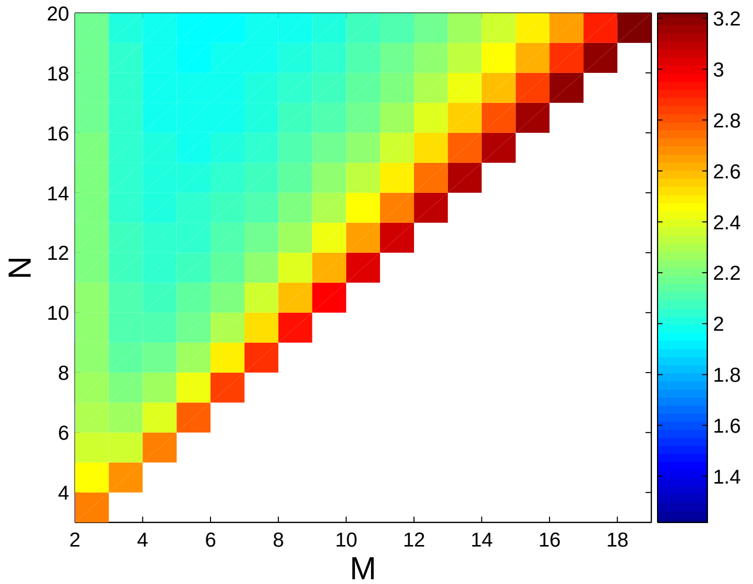

Figure 6: Impact of the size of the merger on the relative gain from the second mover advantage $\Delta(M, N)$. The parameters are $(r, \mu, \sigma, K) = (0.2, 0.15, 0.4, 1)$ and $\gamma = 5$.

4 Mathematical Appendix

4.1 Solution to the price game: case of three firms

$$
\begin{aligned}
q_{it} &= \sqrt{Y_t} - p_{it} - \gamma\Big(p_{it} - \frac{p_{it}+p_{jt}+p_{kt}}{3}\Big), \quad \pi_{it} = p_{it}q_{it}\\
q_{jt} &= \sqrt{Y_t} - p_{jt} - \gamma\Big(p_{jt} - \frac{p_{it}+p_{jt}+p_{kt}}{3}\Big), \quad \pi_{jt} = p_{jt}q_{jt}\\
q_{kt} &= \sqrt{Y_t} - p_{kt} - \gamma\Big(p_{kt} - \frac{p_{it}+p_{jt}+p_{kt}}{3}\Big), \quad \pi_{kt} = p_{kt}q_{kt}.
\end{aligned}
$$

To ease the presentation of results we assume that p_{it}, p_{jt}, p_{kt} are linear transformations of the shock process $\sqrt{Y_t}$, i.e. $p_{it} = a_i\sqrt{Y_t}$, $p_{jt} = a_j\sqrt{Y_t}$, and $p_{kt} = a_k\sqrt{Y_t}$, where $a_i > 0$, $a_j > 0$ and $a_k > 0$.

Let firms i and j be the coalition members. At each instant in time t, the FOC yielding the maximization of profits is as follows:

$$
\begin{bmatrix} \frac{\partial(\pi_{it}+\pi_{jt})}{\partial a_i} \\ \frac{\partial(\pi_{it}+\pi_{jt})}{\partial a_j} \\ \frac{\partial \pi_{kt}}{\partial a_k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix}
1 - a_i - \gamma\Big(a_i - \frac{a_i+a_j+a_k}{3}\Big) - \Big(1+\frac{2}{3}\gamma\Big)a_i + \frac{1}{3}\gamma a_j \\
1 - a_j - \gamma\Big(a_j - \frac{a_i+a_j+a_k}{3}\Big) - \Big(1+\frac{2}{3}\gamma\Big)a_j + \frac{1}{3}\gamma a_i \\
1 - a_k - \gamma\Big(a_k - \frac{a_i+a_j+a_k}{3}\Big) - \Big(1+\frac{2}{3}\gamma\Big)a_k
\end{bmatrix} Y_t = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

From the first two equations it follows that

$$
2(a_j - a_i)(1+\gamma)Y_t = 0 \Rightarrow a_i = a_j \Rightarrow p_{it} = p_{jt} \quad \text{if } Y_t > 0.
$$

Let $p_{Lt} := p_{it} = p_{jt}$, where $p_{Lt} = a_L\sqrt{Y_t}$ and $p_{Ft} := p_{kt}$ with $p_{Ft} = a_F\sqrt{Y_t}$ be the price charged by the coalition members and by the outsider, respectively. The FOC can be simplified to

$$
\begin{bmatrix}
1 - a_L - \gamma\Big(a_L - \frac{2a_L+a_F}{3}\Big) - \Big(1+\frac{2}{3}\gamma\Big)a_L + \frac{1}{3}\gamma a_L \\
1 - a_F - \gamma\Big(a_F - \frac{2a_L+a_F}{3}\Big) - \Big(1+\frac{2}{3}\gamma\Big)a_F
\end{bmatrix} Y_t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

If $Y_t > 0$ then it must be the case that

$$\begin{bmatrix} -2\Big(1+\frac{\gamma}{3}\Big) & \frac{\gamma}{3} \\ \frac{2\gamma}{3} & -2\Big(1+\frac{2\gamma}{3}\Big) \end{bmatrix}\begin{bmatrix} a_L \\ a_F \end{bmatrix} = -\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The unique solution to this linear system implies the corresponding equilibrium prices as follows:

$$\begin{aligned} p_{Lt} =& a_L\sqrt{Y_t} = \frac{2\Big(1+\frac{2\gamma}{3}\Big)+\frac{\gamma}{3}}{4\Big(1+\frac{\gamma}{3}\Big)\Big(1+\frac{2\gamma}{3}\Big)-\frac{2}{9}\gamma^2}\sqrt{Y_t} = \frac{6+5\gamma}{12+12\gamma+2\gamma^2}\sqrt{Y_t} \\ p_{Ft} =& a_F\sqrt{Y_t} = \frac{2\Big(1+\frac{\gamma}{3}\Big)+\frac{2\gamma}{3}}{4\Big(1+\frac{\gamma}{3}\Big)\Big(1+\frac{2\gamma}{3}\Big)-\frac{2}{9}\gamma^2}\sqrt{Y_t} = \frac{6+4\gamma}{12+12\gamma+2\gamma^2}\sqrt{Y_t} \end{aligned}$$

Analysis of the SOC:

$$H = \begin{bmatrix} \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_i^2} & \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_i \partial a_j} & \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_i \partial a_k} \\ \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_j \partial a_i} & \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_j^2} & \frac{\partial^2(\pi_{it}+\pi_{jt})}{\partial a_j \partial a_k} \\ \frac{\partial^2 \pi_{kt}}{\partial a_k \partial a_i} & \frac{\partial^2 \pi_{kt}}{\partial a_k \partial a_j} & \frac{\partial^2 \pi_{kt}}{\partial a_k^2} \end{bmatrix} = \begin{bmatrix} -2(1+\frac{2}{3}\gamma) & \frac{2}{3}\gamma & \frac{1}{3}\gamma \\ \frac{2}{3}\gamma & -2(1+\frac{2}{3}\gamma) & \frac{1}{3}\gamma \\ \frac{1}{3}\gamma & \frac{1}{3}\gamma & -2(1+\frac{2}{3}\gamma) \end{bmatrix}$$

Let $z := -2(1+\frac{2}{3}\gamma) - \lambda$. Derivation of eigenvalues of H:

$$\det(H-\lambda I) = 0 \quad \Leftrightarrow \quad z^3 - 6z\Big(\frac{1}{3}\gamma\Big)^2 + 4\Big(\frac{1}{3}\gamma\Big)^3 = 0$$

Let $x := 3z\gamma^{-1}$ and let $\phi(x) = x^3 - 6x + 4 = (x-2)(x^2+2x-2)$

$$\det(H-\lambda I) = 0 \quad \Leftrightarrow \quad \Big(\frac{1}{3}\gamma\Big)^3\phi(x) = 0$$

Roots of $\phi(x)$ are $-1-\sqrt{3}$, $-1+\sqrt{3}$, and 2. From definition of z and x it follows that $\lambda(x) = -2\Big(1+\frac{2}{3}\gamma\Big) - x\frac{1}{3}\gamma$. Trivially, $\lambda(2) < 0$ and $\lambda(-1+\sqrt{3}) < 0$. It remains to examine

$$\lambda(-1-\sqrt{3}) = -2\Big(1+\frac{2}{3}\gamma\Big) + (1+\sqrt{3})\frac{1}{3}\gamma = -1 + \frac{\sqrt{3}-4}{3}\gamma < 0.$$

Thus H is negative definite, so the FOC delivers the unique global maximum.

4.2 Derivation of the outsider's value

Let T_L be the hitting time of the set $[\pi^*, \infty)$. Let

$$g(\pi_t) := \mathbb{E}_t \int_t^{T_L} \frac{R_s}{R_t}\pi_s ds.$$

As long as $\pi_t < \pi^*$ we have the following recursive expression for $g(\cdot)$

$$g(\pi_t) = \pi_t + \mathbb{E}_t\Big[\frac{R_{t+dt}}{R_t}\int_{t+dt}^{T_L}\frac{R_s}{R_{t+dt}}\pi_s ds\Big] = \pi_t + \exp(-rdt)\mathbb{E}_t[g(\pi_t + d\pi_t)]$$

By using Taylor expansion for $\exp(-rdt)$ around 0 and by application of Itô lemma inside the bracket, and eventually by letting $dt \downarrow 0$ we obtain the following differential equation

$$\frac{\sigma^2}{2}g_{\pi,\pi}\pi^2 + \mu g_\pi \pi - rg + \pi = 0. \tag{2}$$

Solving (2) wrt the boundary conditions $g(\pi^*) = 0$ and $g(0) = 0$ results in

$$g(\pi_t) = -\frac{\pi^*}{r-\mu}\Big(\frac{\pi_t}{\pi^*}\Big)^\beta + \frac{\pi_t}{r-\mu}.$$

Having established the result above, the value of the outsider's brand can be expressed as follows

$$F(\pi_t) = \mathbb{E}_t\int_t^\infty \frac{R_s}{R_t}\frac{D_F}{D}\pi_s ds - \mathbb{E}_t\int_t^{T_L}\frac{R_s}{R_t}\frac{D_F - D}{D}\pi_s ds = \frac{\pi_t}{r-\mu} + \frac{D_F - D}{D}\frac{\pi^*}{r-\mu}\Big(\frac{\pi_t}{\pi^*}\Big)^\beta.$$

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